The Ratio of Fluids to Solid Temperature and/ or Concentration in Fixed-bed Processes

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Design calculations related to heat transfer rates in fixed beds frequently involve the assumption that local values of solid and gas temperatures are equal. For the parallel mass transfer problem (for example, adsorption or ion exchange) gas and solid concentrations are often assumed to be directly proportional. It is obviously desirable to examine this assumption to determine under what conditions the ratio of fluid to solid temperature or concentration is negligibly different from unity. For at least one class of problems this question can easily be answered with information and techniques currently available in the technical literature.

Goldstein (*1, 2*) has developed the mathematics of wave-propagation rates through a fixed bed where there is no transfer across the containing wall, but a nonlinear surface rate equation is used; for example, in the case of adsorption, the latter corresponds to the use of a Langmuir type of isotherm. His results may be applied directly to determine the ratio of the solid to fluid concentration. The mathematical development presented by Goldstein is both lengthy and complex; however the useful results for the engineering student or for the research or design engineer are summarized below.

$$u=\frac{J(rx,\,y)}{e^{(1-r)(x-y)}\{1-J(x,\,ry)\}+J(rx,\,y)} \tag{1}$$

$$v=\frac{1-J(y,\,rx)}{e^{(1-r)(x-y)}\{1-J(x,\,ry)\}+J(rx,\,y)} \tag{2}$$

where

$$J(x,\,y)\equiv 1-e^{-y}\int_0^x e^{-s}I_0(2\sqrt{sy})\,ds \tag{3}$$

and

	For adsorption	*For heat transfer*
$u =$	$\frac{C_A - C_{A_0}{}^*}{C_{A_0} - C_{A_0}{}^*}$	$\frac{T_f - T_{s_0}}{T_{f_0} - T_{s_0}}$
$v =$	$\frac{q_A - q_{A_0}}{q_{A_0}^* - q_{A_0}}$	$\frac{T_s - T_{s_0}}{T_{f_0} - T_{s_0}}$
$r =$	$\frac{1}{1 + K_A C_{A_0}}$	1
$x =$	$\frac{k\rho_B\rho_f q^* z}{G}$	$= \frac{h}{c_{p_f}}\frac{V}{GA}$
$y =$	$\frac{k(1 + K_A C_{A_0})\alpha\rho_f}{KG}\left(\frac{G}{\alpha\rho_f}\,t - z\right)$	$= \frac{h}{C_{p_s}(1-\alpha)}\left(t - \frac{\alpha\rho_f z}{G}\right)$

In this development the kinetic equation for the rate of adsorption by the solid is

$$\frac{\partial q}{\partial t} = k\left[C(q^* - q) - \frac{q}{K}\right]$$

This corresponds to the use of a Langmuir type of adsorption isotherm, that is

$$C^* = \frac{q/q^*}{K(1 - q/q^*)} = \frac{q}{K(q^* - q)}$$

with a rate equation for the surface adsorption expressed as

$$\frac{\partial q}{\partial t} = k(C - C^*)$$

Thus the ratio of fluid to solids temperature, for example, at any time and bed location may be calculated:

$$\frac{u}{v} \equiv \frac{T_f - T_{s_0}}{T_s - T_{s_0}} = \frac{J(x,\,y)}{1 - J(y,\,x)}$$

and, correspondingly, the ratio of fluid to solid concentration may be calculated:

$$\frac{u}{v} \equiv \frac{C_A - C_{A_0}{}^*}{q_A - q_{A_0}} = \frac{J(rx,\,y)}{1 - J(y,\,rx)}$$

To apply this method to the case of adsorption or ion exchange will require values of r and hence will require knowledge of the adsorption rate, data that are available in the literature. Thus the assumption regarding equality of fluid and solid concentrations can be checked.

Opler and Hiester (*3*) have prepared extensive tables of values for both u and v as a function of x and with parameters of v/x. From these tables values can be obtained for any specific problem of heating a fixed bed of solid particles with a hot gas from some initial temperature to an arbitrary final temperature. The ratio of u/v as a function of elapsed time for various values of bed length x is shown in Figure 1. This method may be applied to a specific problem as follows.

A bed is considered which is 4 in. in diameter, 2 ft. in length; a gas flow rate is 150 cu. ft./hr. at the inlet temperature, and particle size is approximately ⅛ in. A value of the convection heat-transfer coefficient of approximately ($h =$) 100 B.t.u./(hr.) (sq. ft./°F.) is calculated from standard equations in the chemical engineering literature. Values for solid- and gas-phase heat capacity and densities were taken from the handbook, and a fraction void of 0.4 was assumed. For an elapsed time of 1 hr. it was found that

$$\frac{u}{v} \simeq \frac{0.1}{0.07} \sim 1.44$$

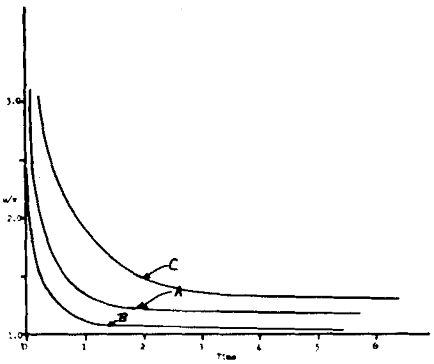

Fig. 1. Fluid/solid temperature ratio vs. elapsed time. Curve A : $X = 18$, B : $X = 25$, C : $X = 10$, time in hours.

NOTATION

A = cross-sectional area of bed = L^2
C = fluid phase concentration = M/L^3
c_p = heat capacity = heat units/M, °F.
G = mass rate of flow, $M/L^2\theta$
h = convection heat transfer coefficient (the analogue of k) = heat units/ $L^3\theta$, °F.
k = adsorption rate constant = $L^3/\theta M$
K = adsorption equilibrium constant = L^3/M
q = solid phase concentration = M/g.
t = time
T = temperature
V = bed volume
z = bed length = L

Greek Letters

α = fraction void
ρ = density = M/L^3

Subscripts

0 = initial conditions
f = fluid
s = solid
B = bed

Superscript

* = equilibrium

LITERATURE CITED

1. Goldstein, S., *Proc. Roy. Soc.* (*London*), **A219**, 151 (1953).
2. *Ibid.*, 171.
3. Opler, Ascher, and N. K. Heister, "Tables for Predicting the Performance of Fixed Bed Ion Exchange and Similar Mass Transfer Processes," Stanford Research Institute, Stanford, Calif. (1954).